Tchebyshefff's Thm

$Y \sim (\mu, \sigma^2)$ ← any dist

$$P(\mu - k\sigma < Y < \mu + k\sigma) \geq 1 - \frac{1}{k^2}$$

N.B.

Proof

$$\sigma^2 = \int_{-\infty}^{\infty} (y-\mu)^2 f_Y(y)\, dy$$

all 3 ⊕ b.c. $(y-\mu)^2$ is ⊕ and f_Y is ⊕

$$= \int_{-\infty}^{\mu - k\sigma} \cdots dy + \int_{\mu - k\sigma}^{\mu + k\sigma} \cdots dy + \int_{\mu + k\sigma}^{\infty}$$

$$\geq \int_{-\infty}^{\mu - k\sigma} \cdots dy + \int_{\mu + k\sigma}^{\infty} \cdots dy$$

$$\int_{-\infty}^{\mu - k\sigma} (y-\mu)^2 f_Y(y)\, dy$$

$$y \leq \mu - k\sigma$$

$2 \geq -5$

$4 \geq 25$

$$y - \mu \leq -k\sigma$$
$$\mu - y \geq k\sigma$$
$$(\mu - y)^2 \geq (k\sigma)^2$$
$$(\mu - y)^2 f_Y(y) \geq (k\sigma)^2 f_Y(y)$$

$$\int_{-\infty}^{\mu - k\sigma} (\mu - y)^2 f_Y(y)\, dy \geq \int_{-\infty}^{\mu - k\sigma} (k\sigma)^2 f_Y(y)\, dy$$

$$-4 \leq 2$$
$$16 \not\leq 4$$

Bivariate Mass Function

- e.g. 2 dice

$$p(d_1 = a \text{ and } d_2 = b) = \frac{1}{36}$$

$$1 \le a, b \le 6$$

$$p(y) = P(Y = y)$$

$$F(y) = P(Y \le y)$$

$$F_{Y_1, Y_2}(y_1, y_2) = P(Y_1 \le y_1, \; Y_2 \le y_2)$$

$$F_{Y_1, Y_2}(y_1, y_2) = \sum_{t_1 \le y_1} \sum_{t_2 \le y_2} p_{Y_1}(t_1, t_2)$$

Continuous

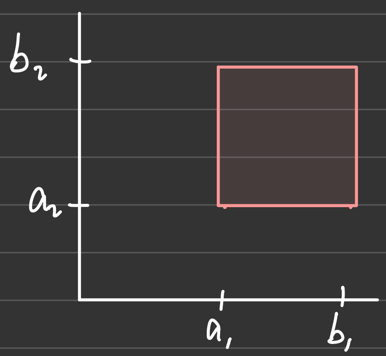
$$F_{Y_1, Y_2}(y_1, y_2) = P(Y_1 < y_1 \text{ and } Y_2 < y_2)$$

$$= \int_{-\infty}^{y_2} \int_{-\infty}^{y_1} f_{Y_1, Y_2}(t_1, t_2)\, dt_1\, dt_2$$

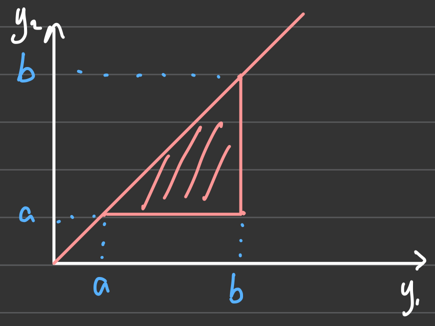
N.B.

$$P(a_1 \le Y_1 \le b_1, \; a_2 \le Y_2 \le b_2)$$

$$= F_{Y_1, Y_2}(b_1, b_2) - F_{Y_1, Y_2}(a_1, b_2) - F_{Y_1, Y_2}(b_1, a_2) + F_{Y_1, Y_2}(a_1, a_2)$$



let $p = P(a \le Y_1 \le Y_2 \le b)$



$$p = \int_a^b \int_a^{y_1} f(y_1, y_2)\, dy_2\, dy_1$$

Marginal density (ie. f_{Y_1}, f_{Y_2})

$$f_{Y_1}(y_1) = \int_{-\infty}^{\infty} f(y_1, y_2)\, dy_2$$

$*$ Always give domain

$$f_{Y_2}(y_2) = \int_{-\infty}^{\infty} f(y_1, y_2)\, dy_1$$

Conditional density

$$F_{Y_1|Y_2}(y_1|y_2) = P(Y_1 < y_1 \mid Y_2 < y_2)$$

$$= \frac{P(Y_1 < y_1 \cap Y_2 < y_2)}{P(Y_2 < y_2)}$$